**Pdf 176 -196**

**Overview of Last Class**

Logical reasoning forms the basis for a huge domain of computer science and mathematics. They help in establishing mathematical arguments, valid or invalid.

**Propositional Logic:**

A proposition is basically a declarative sentence that has a truth value. Truth value can either be true or false, but it needs to be assigned any of the two values and not be ambiguous. The purpose of using propositional logic is to analyze a statement, individually or compositely.

* (a+b)2 = a2 + 2ab + b2
* If x is real, then x2 >= 0
* If x is real, then x2 < 0
* The sun rises in the east.
* The sun rises in the west.

Are all propositions because they have a specific truth value, true or false.

The branch of logic that deals with proposition is propositional logic. Using propositional logic is to analyze a statement, individually or compositely.

**First-Order Logic in Artificial intelligence**

In the topic of Propositional logic, we have seen that how to represent statements using propositional logic. But unfortunately, in propositional logic, we can only represent the facts, which are either true or false. PL is not sufficient to represent the complex sentences or natural language statements. The propositional logic has very limited expressive power. Consider the following sentence, which we cannot represent using PL logic.

*"Some humans are intelligent", or*

*"Sachin likes cricket."*

To represent the above statements, PL logic is not sufficient, so we required some more powerful logic, such as first-order logic.

**First-Order logic:**

First-order logic is another way of knowledge representation in artificial intelligence. It is an extension to propositional logic.

FOL is sufficiently expressive to represent the natural language statements in a concise way.

First-order logic is also known as Predicate logic or First-order predicate logic. First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.

First-order logic (like natural language) does not only assume that the world contains facts like propositional logic but also assumes the following things in the world:

1. **Objects:** A, B, people, numbers, colors, wars, theories, squares, pits.
2. **Relations:** It can be unary relation such as: red, round, is adjacent, or n-any relation such as: the sister of, brother of, has color, comes between
3. **Function:** Father of, best friend, third inning of, end of,

As a natural language, first-order logic also has two main parts:

1. Syntax
2. Semantics

**Syntax of First-Order logic:**

The syntax of FOL determines which collection of symbols is a logical expression in first-order logic. The basic syntactic elements of first-order logic are symbols. We write statements in short-hand notation in FOL.

**Basic Elements of First-order logic:**

Following are the basic elements of FOL syntax:

|  |  |
| --- | --- |
| Constant | 1, 2, A, John, Mumbai, cat |
| Variables | x, y, z, a, b |
| Predicates | Brother, Father, > |
| Function | sqrt, LeftLegOf |
| Connectives | ∧, ∨, ¬, ⇒, ⇔ |
| Equality | == |
| Quantifier | ∀, ∃ |

**Atomic sentences:**

Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.

We can represent atomic sentences as Predicate (term1, term2, ......, term n).

Example:

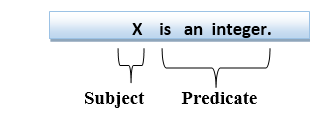
* Ravi and Ajay are brothers: => Brothers (Ravi, Ajay)
* Chinky is a cat: => cat (Chinky)

**Complex Sentences:**

Complex sentences are made by combining atomic sentences using connectives. First-order logic statements can be divided into two parts:

1. **Subject:** Subject is the main part of the statement.
2. **Predicate:** A predicate can be defined as a relation, which binds two atoms together in a statement.

*Consider the statement:* "x is an integer", it consists of two parts, the first part x is the subject of the statement and second part "is an integer," is known as a predicate.



**Quantifiers in First-order logic:**

A quantifier is a language element which generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.

These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression. There are two types of quantifier:

1. **Universal Quantifier,** (for all, everyone, everything)
2. **Existential quantifier**, (for some, at least one).

**Universal Quantifier:**

Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.

The Universal quantifier is represented by a symbol ∀, which resembles an inverted A.

**E.g.,** if x is a variable, then ∀x is read as:

* For all x
* For each x
* For every x

**Existential Quantifier:**

Existential quantifiers are the type of quantifiers, which express that the statement within its scope is true for at least one instance of something.

It is denoted by the logical operator ∃, which resembles as inverted E. When it is used with a predicate variable then it is called as an existential quantifier.

If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:

* There exists a 'x'
* For some 'x'
* For at least one 'x'

**Points to remember:**

1. The main connective for universal quantifier ∀ is implication →.
2. The main connective for existential quantifier ∃ is and ∧.

**Some Examples of FOL using quantifier:**

1. **All birds fly**

In this question the predicate is "fly (bird)."

And since there are all birds who fly so it will be represented as follows.

* 1. ∀x bird(x) →fly(x).

1. **Every man respects his parent.**

In this question, the predicate is "respect(x, y)," where x=man, and y= parent.

Since there is every man so will use ∀, and it will be represented as follows:

* 1. ∀x man(x) → respects (x, parent).

1. **Some boys play cricket**

In this question, the predicate is "play(x, y)," where x= boys, and y= game. Since there are some boys so we will use ∃, and it will be represented as:

* 1. ∃x boys(x) → play(x, cricket).

1. **Not all students like both Mathematics and Science**

In this question, the predicate is "like(x, y)," where x= student, and y= subject.

Since there are not all students, so we will use ∀ with negation, so following representation for this:

* 1. ¬∀ (x) [ student(x) → like(x, Mathematics) ∧ like(x, Science)].

|  |  |  |
| --- | --- | --- |
|  | **Propositional Logic** | **Predicate Logic** |
| **1** | Propositional logic is the logic that deals with a collection of declarative statements which have a truth value, true or false. | Predicate logic is an expression consisting of variables with a specified domain. It consists of objects, relations and functions between the objects. |
| **2** | It is the basic and most widely used logic. Also known as Boolean logic. | It is an extension of propositional logic covering predicates and quantification. |
| **3** | A proposition has a specific truth value, either true or false. | A predicate’s truth value depends on the variables’ value. |
| **4** | Scope analysis is not done in propositional logic. | Predicate logic helps analyze the scope of the subject over the predicate. There are three quantifiers: Universal Quantifier (∀) depicts for all, Existential Quantifier (∃) depicting there exists some and Uniqueness Quantifier (∃!) depicting exactly one. |
| **5** | Propositions are combined with Logical Operators or Logical Connectives like Negation (¬), Disjunction (∧), Conjunction (∨), Exclusive OR (⊕), Implication (⇒), Bi-Conditional or Double Implication (⇔). | Predicate Logic adds by introducing quantifiers to the existing proposition. |
| **6** | It is a more generalized representation. | It is a more specialized representation. |
| **7** | It cannot deal with sets of entities. | It can deal with set of entities with the help of quantifiers. |